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# **matrix\_decomposition Documentation**

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This is *matrix-decomposition*, a library to approximate Hermitian (dense and sparse) matrices by positive definite matrices. Furthermore it allows to decompose (factorize) positive definite matrices and solve associated systems of linear equations.



# CHAPTER 1

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## Release info

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There are several ways to obtain and install this package.

### 1.1 Conda

To install this package with *conda* run:

```
conda install -c jore matrix-decomposition  
https://anaconda.org/jore/matrix-decomposition
```

### 1.2 pip

To install this package with *pip* run:

```
pip install 'matrix-decomposition'  
https://pypi.python.org/pypi/matrix-decomposition
```

## 1.3 GitHub

To clone this package with *git* run:

```
git clone https://github.com/jor-/matrix-decomposition.git
```

To install this package after that with *python* run:

```
cd matrix-decomposition; python setup.py install
```

```
https://github.com/jor-/matrix-decomposition
```

## 1.4 Zenodo

# CHAPTER 2

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## Documentation

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<https://matrix-decomposition.readthedocs.io>



## CHAPTER 3

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Test status

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# CHAPTER 4

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## Contents

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## 4.1 Functions

Several functions are included in this package. The most important ones are summarized here.

### 4.1.1 Positive semidefinite approximation of a matrix

```
matrix.approximation.positive_semidefinite.positive_semidefinite_matrix(A,  
                           min_diag_B=None,  
                           max_diag_B=None,  
                           min_diag_D=None,  
                           max_diag_D=None,  
                           min_abs_value_D=None,  
                           min_abs_value_L=None,  
                           per-  
                           mu-  
                           ta-  
                           tion=None,  
                           over-  
                           write_A=False)
```

Computes an approximation of  $A$  which has a  $LDL^H$  decomposition with the specified properties.

Returns  $A$  if  $A$  has such a decomposition and otherwise an approximation of  $A$ .

#### Parameters

- **A** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be approximated.  $A$  must be Hermitian.
- **min\_diag\_B** (`numpy.ndarray` or `float`) – Each component of the diagonal of the returned matrix is forced to be greater or equal to `min_diag_B`. optional, default : No minimal value is forced.

- **max\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the returned matrix is forced to be lower or equal to `max_diag_B`. optional, default : No maximal value is forced.
- **min\_diag\_D** (`float`) – Each component of the diagonal of the matrix  $D$  in a  $LDL^H$  decomposition of the returned matrix is forced to be greater or equal to `min_diag_D`. `min_diag_D` must be greater or equal to 0. optional, default : Is chosen by the algorithm.
- **max\_diag\_D** (`float`) – Each component of the diagonal of the matrix  $D$  in a  $LDL^H$  decomposition of the returned matrix is forced to be lower or equal to `max_diag_D`. optional, default : No maximal value is forced.
- **min\_abs\_value\_D** (`float`) – Absolute values below `min_abs_value_D` are considered as zero in the matrix  $D$  in a  $LDL^H$  decomposition of the returned matrix. `min_abs_value_D` must be greater or equal to 0. optional, default : The square root of the resolution of the underlying data type.
- **min\_abs\_value\_L** (`float`) – Absolute values below `min_abs_value_L` are considered as zero in the matrix  $L$  of an approximated  $LDL^H$  decomposition. `min_abs_value_L` must be greater or equal to 0. optional, default : The resolution of the underlying data type.
- **permutation** (`str or numpy.ndarray`) – The symmetric permutation method that is applied to the matrix before it is decomposed. It has to be a value in `matrix.UNIVERSAL_PERMUTATION_METHODS` or `APPROXIMATION_ONLY_PERMUTATION_METHODS`. If  $A$  is sparse, it can also be a value in `matrix.SPARSE_ONLY_PERMUTATION_METHODS`. It is also possible to directly pass a permutation vector. optional, default: The permutation is chosen by the algorithm.
- **overwrite\_A** (`bool`) – Whether it is allowed to overwrite  $A$ . Enabling may result in performance gain. optional, default: False

**Returns** **B** – An approximation of  $A$  which has a  $LDL^H$  decomposition.

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix` (same type as  $A$ )

**Raises**

- `matrix.errors.MatrixNotSquareError` – If  $A$  is not a square matrix.
- `matrix.errors.MatrixComplexDiagonalValueError` – If  $A$  has complex diagonal values.

```
matrix.approximation.positive_semidefinite.decomposition(A, min_diag_B=None,
                                                       max_diag_B=None,
                                                       min_diag_D=None,
                                                       max_diag_D=None,
                                                       min_abs_value_D=None,
                                                       min_abs_value_L=None,
                                                       permutation=None,
                                                       overwrite_A=False,
                                                       return_type=None)
```

Computes an approximative decomposition of a matrix with the specified properties.

Returns a decomposition of  $A$  if has such a decomposition and otherwise a decomposition of an approximation of  $A$ .

**Parameters**

- **A** (`numpy.ndarray or scipy.sparse.spmatrix`) – The matrix that should be approximated by a decomposition.  $A$  must be Hermitian.

- **min\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the composed matrix  $B$  of an approximated  $LDL^H$  decomposition is forced to be greater or equal to  $\text{min\_diag\_B}$ . optional, default : No minimal value is forced.
- **max\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the composed matrix  $B$  of an approximated  $LDL^H$  decomposition is forced to be lower or equal to  $\text{max\_diag\_B}$ . optional, default : No maximal value is forced.
- **min\_diag\_D** (`float`) – Each component of the diagonal of the matrix  $D$  in an approximated  $LDL^H$  decomposition is forced to be greater or equal to  $\text{min\_diag\_D}$ .  $\text{min\_diag\_D}$  must be greater or equal to 0. optional, default : Is chosen by the algorithm.
- **max\_diag\_D** (`float`) – Each component of the diagonal of the matrix  $D$  in an approximated  $LDL^H$  decomposition is forced to be lower or equal to  $\text{max\_diag\_D}$ . optional, default : No maximal value is forced.
- **min\_abs\_value\_D** (`float`) – Absolute values below  $\text{min\_abs\_value\_D}$  are considered as zero in the matrix  $D$  of an approximated  $LDL^H$  decomposition.  $\text{min\_abs\_value\_D}$  must be greater or equal to 0. optional, default : The square root of the resolution of the underlying data type.
- **min\_abs\_value\_L** (`float`) – Absolute values below  $\text{min\_abs\_value\_L}$  are considered as zero in the matrix  $L$  of an approximated  $LDL^H$  decomposition.  $\text{min\_abs\_value\_L}$  must be greater or equal to 0. optional, default : The resolution of the underlying data type.
- **permutation** (`str or numpy.ndarray`) – The symmetric permutation method that is applied to the matrix before it is decomposed. It has to be a value in `matrix.UNIVERSAL_PERMUTATION_METHODS` or `APPROXIMATION_ONLY_PERMUTATION_METHODS`. If  $A$  is sparse, it can also be a value in `matrix.SPARSE_ONLY_PERMUTATION_METHODS`. It is also possible to directly pass a permutation vector. optional, default: The permutation is chosen by the algorithm.
- **overwrite\_A** (`bool`) – Whether it is allowed to overwrite  $A$ . Enabling may result in performance gain. optional, default: False
- **return\_type** (`str`) – The type of the decomposition that should be returned. It has to be a value in `matrix.DECOMPOSITION_TYPES`. optional, default : The type of the decomposition is chosen by the function itself.

**Returns** An (approximative) decomposition of  $A$  of type `return_type`.

**Return type** `matrix.decompositions.DecompositionBase`

**Raises**

- `matrix.errors.MatrixNotSquareError` – If  $A$  is not a square matrix.
- `matrix.errors.MatrixComplexDiagonalValueError` – If  $A$  has complex diagonal values.

```
matrix.approximation.positive_semidefinite.APPROXIMATION_ONLY_PERMUTATION_METHODS = ('minin'
Permutation methods supported only by the decomposition and the positive_semidefinite_matrix algorithm.
```

```
matrix.approximation.positive_semidefinite.GMW_81(A, min_diag_B=None,
                                                    max_diag_B=None,
                                                    min_diag_D=None, over-
                                                    write_A=False)
```

Computes a positive definite approximation of  $A$ .

Returns  $A$  if  $A$  is positive definite and meets the constrains and otherwise a positive definite approximation of  $A$ .

**Parameters**

- **A** (`numpy.ndarray`) – The matrix that should be approximated.  $A$  must be symmetric.
- **min\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the returned matrix is forced to be greater or equal to  $\min_{diag} B$ . optional, default : No minimal value is forced.
- **max\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the returned matrix is forced to be lower or equal to  $\max_{diag} B$ . optional, default : No maximal value is forced.
- **min\_diag\_D** (`float`) – Each component of the diagonal of the matrix  $D$  in a  $LDL^T$  decomposition of the returned matrix is forced to be greater or equal to  $\min_{diag} D$ .  $\min_{diag} D$  must be positive. optional, default : Is chosen by the algorithm.
- **overwrite\_A** (`bool`) – Whether it is allowed to overwrite  $A$ . Enabling may result in performance gain. optional, default: False

**Returns** **B** – An approximation of  $A$  which is positive definite.

**Return type** `numpy.ndarray`

**Raises** `matrix.errors.MatrixNotSquareError` – If  $A$  is not a square matrix.

## Notes

The algorithm is introduced in [1]. It is also described in [2]. This description has been used for this implementation. The implementation has been extended to allow restrictions on the diagonal values.

## References

- [1] Gill, P. E.; Murray, W. & Wright, M. H., Practical optimization, Academic press, 1981
- [2] Fang, H.-r. & O'Leary, D. P., Modified Cholesky algorithms: a catalog with new approaches, Mathematical Programming, 2008, 115, 319-349

```
matrix.approximation.positive_semidefinite.GMW_T1 (A, min_diag_B=None,
                                                    max_diag_B=None,
                                                    min_diag_D=None,      over-
                                                    write_A=False)
```

Computes a positive definite approximation of  $A$ .

Returns  $A$  if  $A$  is positive definite and meets the constraints and otherwise a positive definite approximation of  $A$ .

## Parameters

- **A** (`numpy.ndarray`) – The matrix that should be approximated.  $A$  must be symmetric.
- **min\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the returned matrix is forced to be greater or equal to  $\min_{diag} B$ . optional, default : No minimal value is forced.
- **max\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the returned matrix is forced to be lower or equal to  $\max_{diag} B$ . optional, default : No maximal value is forced.
- **min\_diag\_D** (`float`) – Each component of the diagonal of the matrix  $D$  in a  $LDL^T$  decomposition of the returned matrix is forced to be greater or equal to  $\min_{diag} D$ .  $\min_{diag} D$  must be positive. optional, default : Is chosen by the algorithm.

- **overwrite\_A** (`bool`) – Whether it is allowed to overwrite  $A$ . Enabling may result in performance gain. optional, default: False

**Returns**  $B$  – An approximation of  $A$  which is positive definite.

**Return type** `numpy.ndarray`

**Raises** `matrix.errors.MatrixNotSquareError` – If  $A$  is not a square matrix.

## Notes

The algorithm is introduced in [1]. This description has been used for this implementation. The algorithm is based on [2]. The implementation has been extended to allow restrictions on the diagonal values.

## References

[1] Fang, H.-r. & O’Leary, D. P., Modified Cholesky algorithms: a catalog with new approaches, Mathematical Programming, 2008, 115, 319-349

[2] Gill, P. E.; Murray, W. & Wright, M. H., Practical optimization, Academic press, 1981

```
matrix.approximation.positive_semidefinite.GMW_T2 (A, min_diag_B=None,  
                                                 max_diag_B=None,  
                                                 min_diag_D=None, over-  
                                                 write_A=False)
```

Computes a positive definite approximation of  $A$ .

Returns  $A$  if  $A$  is positive definite and meets the constraints and otherwise a positive definite approximation of  $A$ .

## Parameters

- **A** (`numpy.ndarray`) – The matrix that should be approximated.  $A$  must be symmetric.
- **min\_diag\_B** (`numpy.ndarray` or `float`) – Each component of the diagonal of the returned matrix is forced to be greater or equal to  $\text{min\_diag\_B}$ . optional, default : No minimal value is forced.
- **max\_diag\_B** (`numpy.ndarray` or `float`) – Each component of the diagonal of the returned matrix is forced to be lower or equal to  $\text{max\_diag\_B}$ . optional, default : No maximal value is forced.
- **min\_diag\_D** (`float`) – Each component of the diagonal of the matrix  $D$  in a  $LDL^T$  decomposition of the returned matrix is forced to be greater or equal to  $\text{min\_diag\_D}$ .  $\text{min\_diag\_D}$  must be positive. optional, default : Is chosen by the algorithm.
- **overwrite\_A** (`bool`) – Whether it is allowed to overwrite  $A$ . Enabling may result in performance gain. optional, default: False

**Returns**  $B$  – An approximation of  $A$  which is positive definite.

**Return type** `numpy.ndarray`

**Raises** `matrix.errors.MatrixNotSquareError` – If  $A$  is not a square matrix.

## Notes

The algorithm is introduced in [1]. This description has been used for this implementation. The algorithm is based on [2]. The implementation has been extended to allow restrictions on the diagonal values.

## References

[1] Fang, H.-r. & O'Leary, D. P., Modified Cholesky algorithms: a catalog with new approaches, Mathematical Programming, 2008, 115, 319-349

[2] Gill, P. E.; Murray, W. & Wright, M. H., Practical optimization, Academic press, 1981

```
matrix.approximation.positive_semidefinite.SE_90(A, min_diag_B=None,
                                                max_diag_B=None,
                                                min_diag_D=None,          over-
                                                write_A=False)
```

Computes a positive definite approximation of  $A$ .

Returns  $A$  if  $A$  is positive definite and meets the constraints and otherwise a positive definite approximation of  $A$ .

### Parameters

- **A** (`numpy.ndarray`) – The matrix that should be approximated.  $A$  must be symmetric.
- **min\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the returned matrix is forced to be greater or equal to  $\text{min\_diag\_B}$ . optional, default : No minimal value is forced.
- **max\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the returned matrix is forced to be lower or equal to  $\text{max\_diag\_B}$ . optional, default : No maximal value is forced.
- **min\_diag\_D** (`float`) – Each component of the diagonal of the matrix  $D$  in a  $LDL^T$  decomposition of the returned matrix is forced to be greater or equal to  $\text{min\_diag\_D}$ .  $\text{min\_diag\_D}$  must be positive. optional, default : Is chosen by the algorithm.
- **overwrite\_A** (`bool`) – Whether it is allowed to overwrite  $A$ . Enabling may result in performance gain. optional, default: False

**Returns** **B** – An approximation of  $A$  which is positive definite.

**Return type** `numpy.ndarray`

**Raises** `matrix.errors.MatrixNotSquareError` – If  $A$  is not a square matrix.

## Notes

The algorithm is introduced in [1]. It is also described in [2]. This description has been used for this implementation. The implementation has been extended to allow restrictions on the diagonal values.

## References

[1] Schnabel, R. & Eskow, E., A New Modified Cholesky Factorization, SIAM Journal on Scientific and Statistical Computing, 1990, 11, 1136-1158

[2] Fang, H.-r. & O'Leary, D. P., Modified Cholesky algorithms: a catalog with new approaches, Mathematical Programming, 2008, 115, 319-349

```
matrix.approximation.positive_semidefinite.SE_99(A, min_diag_B=None,
                                                max_diag_B=None,
                                                min_diag_D=None,          over-
                                                write_A=False)
```

Computes a positive definite approximation of  $A$ .

Returns  $A$  if  $A$  is positive definite and meets the constraints and otherwise a positive definite approximation of  $A$ .

## Parameters

- **A** (`numpy.ndarray`) – The matrix that should be approximated.  $A$  must be symmetric.
- **min\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the returned matrix is forced to be greater or equal to  $\min_{diag\_B}$ . optional, default : No minimal value is forced.
- **max\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the returned matrix is forced to be lower or equal to  $\max_{diag\_B}$ . optional, default : No maximal value is forced.
- **min\_diag\_D** (`float`) – Each component of the diagonal of the matrix  $D$  in a  $LDL^T$  decomposition of the returned matrix is forced to be greater or equal to  $\min_{diag\_D}$ .  $\min_{diag\_D}$  must be positive. optional, default : Is chosen by the algorithm.
- **overwrite\_A** (`bool`) – Whether it is allowed to overwrite  $A$ . Enabling may result in performance gain. optional, default: False

**Returns** **B** – An approximation of  $A$  which is positive definite.

**Return type** `numpy.ndarray`

**Raises** `matrix.errors.MatrixNotSquareError` – If  $A$  is not a square matrix.

## Notes

The algorithm is introduced in [1]. It is also described in [2]. This description has been used for this implementation. The algorithm is based on [3]. The implementation has been extended to allow restrictions on the diagonal values.

## References

- [1] Schnabel, R. & Eskow, E., A Revised Modified Cholesky Factorization Algorithm, SIAM Journal on Optimization, 1999, 9, 1135-1148
- [2] Fang, H.-r. & O'Leary, D. P., Modified Cholesky algorithms: a catalog with new approaches, Mathematical Programming, 2008, 115, 319-349
- [3] Schnabel, R. & Eskow, E., A New Modified Cholesky Factorization, SIAM Journal on Scientific and Statistical Computing, 1990, 11, 1136-1158

```
matrix.approximation.positive_semidefinite.SE_T1(A, min_diag_B=None,  
                                              max_diag_B=None,  
                                              min_diag_D=None,          over-  
                                              write_A=False)
```

Computes a positive definite approximation of  $A$ .

Returns  $A$  if  $A$  is positive definite and meets the constraints and otherwise a positive definite approximation of  $A$ .

## Parameters

- **A** (`numpy.ndarray`) – The matrix that should be approximated.  $A$  must be symmetric.
- **min\_diag\_B** (`numpy.ndarray or float`) – Each component of the diagonal of the returned matrix is forced to be greater or equal to  $\min_{diag\_B}$ . optional, default : No minimal value is forced.

- **max\_diag\_B** (`numpy.ndarray` or `float`) – Each component of the diagonal of the returned matrix is forced to be lower or equal to *max\_diag\_B*. optional, default : No maximal value is forced.
- **min\_diag\_D** (`float`) – Each component of the diagonal of the matrix *D* in a  $LDL^T$  decomposition of the returned matrix is forced to be greater or equal to *min\_diag\_D*. *min\_diag\_D* must be positive. optional, default : Is chosen by the algorithm.
- **overwrite\_A** (`bool`) – Whether it is allowed to overwrite *A*. Enabling may result in performance gain. optional, default: False

**Returns** **B** – An approximation of *A* which is positive definite.

**Return type** `numpy.ndarray`

**Raises** `matrix.errors.MatrixNotSquareError` – If *A* is not a square matrix.

## Notes

The algorithm is introduced in [1]. This description has been used for this implementation. The algorithm is based on [2]. The implementation has been extended to allow restrictions on the diagonal values.

## References

- [1] Fang, H.-r. & O’Leary, D. P., Modified Cholesky algorithms: a catalog with new approaches, Mathematical Programming, 2008, 115, 319-349
- [2] Schnabel, R. & Eskow, E., A Revised Modified Cholesky Factorization Algorithm, SIAM Journal on Optimization, 1999, 9, 1135-1148
- [3] Schnabel, R. & Eskow, E., A New Modified Cholesky Factorization, SIAM Journal on Scientific and Statistical Computing, 1990, 11, 1136-1158

## 4.1.2 Nearest matrix with specific properties

This module provides functions to compute matrices with minimal difference to an input matrix and specific properties.

`matrix.nearest.correlation_matrix(A, max_iterations=1000)`

Computes a correlation matrix closest to *A* in the Frobenius norm. A correlation matrix is a positive semidefinite matrix with ones on the diagonal.

### Parameters

- **A** (`numpy.ndarray`) – *A* must be Hermitian.
- **max\_iterations** (`int`) – The maximal number of iterations used for the calculation.

**Returns** **B** – A correlation matrix closest to *A* in the Frobenius norm.

**Return type** `numpy.ndarray`

**Raises** `TooManyIterationsError` – If the computation needs more iterations than the maximal number of iterations.

## Notes

The method is presented in [1]. For the computation some code of Michael Croucher is used. See this source code for the license.

## References

- [1] Higham, N. J. Computing the Nearest Correlation Matrix—A Problem from Finance IMA J. Numer. Anal., 2002, 22, 329-343

`matrix.nearest.positive_semidefinite_matrix(A, symmetric=False)`

Computes a symmetric positive semidefinite matrix with minimal difference to  $A$  in the Frobenius norm.

### Parameters

- **A** (`numpy.ndarray`) –  $A$  must be Hermitian.
- **symmetric** (`bool`) – Whether  $A$  can be assumed to be symmetric or not. optional, default: False

**Returns** **B** – A symmetric positive semidefinite Hermitian matrix with minimal difference to  $A$  in the Frobenius norm.

**Return type** `numpy.ndarray`

## Notes

The method is presented in [1].

## References

- [1] Higham, N. J. Computing a nearest symmetric positive semidefinite matrix Linear Algebra and its Applications, 1988, 103, 103-118

`matrix.nearest.skew_symmetric_matrix(A)`

Computes a skew-symmetric matrix with minimal difference to  $A$  for any unitarily invariant norm.

### Parameters

**A** (`numpy.ndarray`) –  $A$  must be a square matrix.

**Returns** **B** – A skew-symmetric matrix with minimal difference to  $A$  for any unitarily invariant norm.

**Return type** `numpy.ndarray`

`matrix.nearest.symmetric_matrix(A)`

Computes a symmetric matrix with minimal difference to  $A$  for any unitarily invariant norm.

**Parameters** **A** (`numpy.ndarray`) –  $A$  must be a square matrix.

**Returns** **B** – A symmetric matrix with minimal difference to  $A$  for any unitarily invariant norm.

**Return type** `numpy.ndarray`

## Notes

The optimality is proven in [1].

## References

- [1] Fan, K., & Hoffman, A. (1955). Some Metric Inequalities in the Space of Matrices. Proceedings of the American Mathematical Society, 6(1), 111-116. doi:10.2307/2032662

### 4.1.3 Decompose a matrix

```
matrix.decompose(A, permutation=None, return_type=None, check_finite=True, overwrite_A=False)
```

Computes a decomposition of a matrix.

#### Parameters

- **A** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Matrix to be decomposed. A must be Hermitian.
- **permutation** (`str` or `numpy.ndarray`) – The symmetric permutation method that is applied to the matrix before it is decomposed. It has to be a value in `matrix.UNIVERSAL_PERMUTATION_METHODS`. If A is sparse, it can also be a value in `matrix.SPARSE_ONLY_PERMUTATION_METHODS`. It is also possible to directly pass a permutation vector. optional, default: no permutation
- **return\_type** (`str`) – The type of the decomposition that should be calculated. It has to be a value in `matrix.DECOMPOSITION_TYPES`. If return\_type is None the type of the returned decomposition is chosen by the function itself. optional, default: the type of the decomposition is chosen by the function itself
- **check\_finite** (`bool`) – Whether to check that A contains only finite numbers. Disabling may result in problems (crashes, non-termination) if the inputs do contain infinities or NaNs. Disabling gives a performance gain. optional, default: True
- **overwrite\_A** (`bool`) – Whether it is allowed to overwrite A. Enabling may result in performance gain. optional, default: False

**Returns** A decomposition of A of type `return_type`.

**Return type** `matrix.decompositions.DecompositionBase`

#### Raises

- `matrix.errors.NoDecompositionPossibleError` – If the decomposition of A is not possible.
- `matrix.errors.MatrixNotSquareError` – If A is not a square matrix.
- `matrix.errors.MatrixNotFiniteError` – If A is not a finite matrix and `check_finite` is True.

```
matrix.UNIVERSAL_PERMUTATION_METHODS = ('none', 'decreasing_diagonal_values', 'increasing_
```

Supported permutation methods for decompose dense and sparse matrices.

```
matrix.SPARSE_ONLY_PERMUTATION_METHODS = ()
```

Supported permutation methods only for sparse matrices.

```
matrix.DECOMPOSITION_TYPES = ('LDL', 'LDL_compressed', 'LL')
```

Supported types of decompositions.

### 4.1.4 Examine a matrix

```
matrix.is_positive_semidefinite(A, check_finite=True)
```

Returns whether the passed matrix is positive semi-definite.

## Parameters

- **A** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be checked. A must be Hermitian.
- **check\_finite** (`bool`) – Whether to check that A contain only finite numbers. Disabling may result in problems (crashes, non-termination) if they contain infinities or NaNs. Disabling gives a performance gain. optional, default: True

**Returns** Whether A is positive semi-definite.

**Return type** `bool`

**Raises** `matrix.errors.MatrixNotFiniteError` – If A is not a finite matrix and `check_finite` is True.

`matrix.is_positive_definite(A, check_finite=True)`

Returns whether the passed matrix is positive definite.

## Parameters

- **A** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be checked. A must be Hermitian.
- **check\_finite** (`bool`) – Whether to check that A contain only finite numbers. Disabling may result in problems (crashes, non-termination) if they contain infinities or NaNs. Disabling gives a performance gain. optional, default: True

**Returns** Whether A is positive definite.

**Return type** `bool`

**Raises** `matrix.errors.MatrixNotFiniteError` – If A is not a finite matrix and `check_finite` is True.

`matrix.is_invertible(A, check_finite=True)`

Returns whether the passed matrix is an invertible matrix.

## Parameters

- **A** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be checked. A must be Hermitian and positive semidefinite.
- **check\_finite** (`bool`) – Whether to check that A contain only finite numbers. Disabling may result in problems (crashes, non-termination) if they contain infinities or NaNs. Disabling gives a performance gain. optional, default: True

**Returns** Whether A is invertible.

**Return type** `bool`

**Raises** `matrix.errors.MatrixNotFiniteError` – If A is not a finite matrix and `check_finite` is True.

## 4.1.5 Solve system of linear equations

`matrix.solve(A, b, overwrite_b=False, check_finite=True)`

Solves the equation  $A x = b$  regarding  $x$ .

## Parameters

- **A** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be checked. A must be Hermitian and positive definite.

- **b** (`numpy.ndarray`) – Right-hand side vector or matrix in equation  $A x = b$ . It must hold  $b.shape[0] == A.shape[0]$ .
- **overwrite\_b** (`bool`) – Allow overwriting data in  $b$ . Enabling gives a performance gain. optional, default: False
- **check\_finite** (`bool`) – Whether to check that  $A$  and  $b'$  contain only finite numbers. Disabling may result in problems (crashes, non-termination) if they contain infinities or NaNs. Disabling gives a performance gain. optional, default: True

**Returns** An  $x$  so that  $A x = b$ . The shape of  $x$  matches the shape of  $b$ .

**Return type** `numpy.ndarray`

#### Raises

- `matrix.errors.MatrixNotSquareError` – If  $A$  is not a square matrix.
- `matrix.errors.MatrixNotFiniteError` – If  $A$  is not a finite matrix and `check_finite` is True.
- `matrix.errors.MatrixSingularError` – If  $A$  is singular.

## 4.2 Matrix decompositions

Several matrix decompositions are supported. They are available in `matrix.decompositions`:

### 4.2.1 LL decomposition

**class** `matrix.decompositions.LL_Decomposition(L=None, p=None)`

Bases: `matrix.decompositions.DecompositionBase`

A matrix decomposition where  $LL^H$  is the decomposed (permuted) matrix.

$L$  is a lower triangle matrix with ones on the diagonal. This decomposition is also called Cholesky decomposition.

#### Parameters

- **L** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix  $L$  of the decomposition. optional, If it is not set yet, it must be set later.
- **p** (`numpy.ndarray`) – The permutation vector used for the decomposition. This decomposition is of  $A[p[:, np.newaxis], p[np.newaxis, :]]$  where  $A$  is a matrix. optional, default: no permutation

#### L

The matrix  $L$  of the decomposition.

**Type** `numpy.matrix` or `scipy.sparse.spmatrix`

#### P

The permutation matrix.  $P @ A @ P^T$  is the matrix  $A$  permuted by the permutation of the decomposition

**Type** `scipy.sparse.dok_matrix`

#### append\_block\_decomposition(dec)

Makes a new decomposition where this decompstion and the passed one are appended.

**Parameters** `dec` (`DecompositionBase`) – The decomposition that should be appended to this decomposition.

**Returns** `dec` – A new decomposition where this decomposition and the passed one are appended.  
This desomposition represents the first block and the passed one the second block.

**Return type** `DecompositionBase`

**as\_LDL\_Decomposition()**

**as\_any\_type** (`*type_strs, copy=False`)

Converts this decomposition to any of the passed types.

**Parameters**

- **\*type\_strs** (`str`) – The decomposition types to any of them this this decomposition is converted.
- **copy** (`bool`) – Whether the data of this decomposition should always be copied or only if needed.

**Returns** If the type of this decomposition is not in `type_strs`, a decomposition of type `type_str[0]` is returned which represents the same decomposed matrix as this decomposition. Otherwise this decomposition or a copy of it is returned, depending on `copy`.

**Return type** `matrix.decompositions.DecompositionBase`

**as\_same\_type** (`dec, copy=False`)

Converts the passed decompositions to the same type as this decomposition.

**Parameters**

- **doc** (`DecompositionBase`) – The decomposition that should be converted.
- **copy** (`bool`) – Whether the data of the decomposition that sould be converted should always be copied or only if needed.

**Returns** If the type of the passed decomposition is not same type as this decomposition, a decomposition of this type is returned which represents the same decomposed matrix as the passed decomposition. Otherwise this decomposition or a copy of it is returned, depending on `copy`.

**Return type** `matrix.decompositions.DecompositionBase`

**as\_type** (`type_str, copy=False`)

Converts this decomposition to passed type.

**Parameters**

- **type\_str** (`str`) – The decomposition type to which this decomposition is converted.
- **copy** (`bool`) – Whether the data of this decomposition should always be copied or only if needed.

**Returns** If the type of this decomposition is not `type_str`, a decomposition of type `type_str` is returned which represents the same decomposed matrix as this decomposition. Otherwise this decomposition or a copy of it is returned, depending on `copy`.

**Return type** `matrix.decompositions.DecompositionBase`

**check\_finite** (`check_finite=True`)

Check if this is a decomposition representing a finite matrix.

**Parameters** `check_finite` (`bool`) – Whether to perform this check. default: True

**Raises** `matrix.errors.DecompositionNotFiniteError` – If this is a decomposition representing a non-finite matrix.

**check\_invertible()**

Check if this is a decomposition representing an invertible matrix.

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**composed\_matrix**

The composed matrix represented by this decomposition.

**Type** `numpy.matrix` or `scipy.sparse.spmatrix`

**copy()**

Copies this decomposition.

**Returns** A copy of this decomposition.

**Return type** `matrix.decompositions.DecompositionBase`

**inverse\_matrix\_both\_sides\_multiplication**(*x*, *y*=*None*, *dtype*=*None*)

Calculates the both sides (matrix-matrix or matrix-vector) product  $y.H @ B @ x$ , where  $B$  is the matrix inverse of the composed matrix represented by this decomposition.

**Parameters**

- **x** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ B @ x$ . It must hold `self.n == x.shape[0]`.
- **y** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ B @ x$ . It must hold `self.n == y.shape[0]`. optional, default: If *y* is not passed, *x* is used as *y*.
- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $y.H @ A @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**inverse\_matrix\_right\_side\_multiplication**(*x*, *dtype*=*None*)

Calculates the right side (matrix-matrix or matrix-vector) product  $B @ x$ , where  $B$  is the matrix inverse of the composed matrix represented by this decomposition.

**Parameters**

- **x** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product in the matrix-matrix or matrix-vector  $B @ x$ . It must hold `self.n == x.shape[0]`.
- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $B @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**is\_almost\_equal**(*other*, *rtol*=0.0001, *atol*= $1e-06$ )

Whether this decomposition is close to passed decomposition.

**Parameters**

- **other** (`str`) – The decomposition which to compare to this decomposition.

- **rtol** (*float*) – The relative tolerance parameter.
- **atol** (*float*) – The absolute tolerance parameter.

**Returns** Whether this decomposition is close to passed decomposition.

**Return type** `bool`

**is\_equal** (*other*)

Whether this decomposition is equal to passed decomposition.

**Parameters** **other** (*str*) – The decomposition which to compare to this decomposition.

**Returns** Whether this decomposition is equal to passed decomposition.

**Return type** `bool`

**is\_finite** ()

Returns whether this is a decomposition representing a finite matrix.

**Returns** Whether this is a decomposition representing a finite matrix.

**Return type** `bool`

**is\_invertible** ()

Returns whether this is a decomposition representing an invertible matrix.

**Returns** Whether this is a decomposition representing an invertible matrix.

**Return type** `bool`

**is\_permuted**

Whether this is a decompositon with permutation.

**Type** `bool`

**is\_positive\_definite** ()

Returns whether this is a decomposition of a positive definite matrix.

**Returns** Whether this is a decomposition of a positive definite matrix.

**Return type** `bool`

**is\_positive\_semidefinite** ()

Returns whether this is a decomposition of a positive semi-definite matrix.

**Returns** Whether this is a decomposition of a positive semi-definite matrix.

**Return type** `bool`

**is\_singular** ()

Returns whether this is a decomposition representing a singular matrix.

**Returns** Whether this is a decomposition representing a singular matrix.

**Return type** `bool`

**is\_sparse** ()

Returns whether this is a decomposition of a sparse matrix.

**Returns** Whether this is a decomposition of a sparse matrix.

**Return type** `bool`

**is\_type** (*type\_str*)

Whether this decomposition is of the passed type.

**Parameters** **type\_str** (*str*) – The decomposition type according to which is checked.

**Returns** Whether this is a decomposition of the passed type.

**Return type** bool

**load**(filename)

Loads a decomposition of this type.

**Parameters** filename (str) – Where the decomposition is saved.

**Raises**

- FileNotFoundError – If the files are not found in the passed directory.
- DecompositionInvalidDecompositionTypeFile – If the files contains another decomposition type.

**matrix\_both\_sides\_multiplication**(x, y=None, dtype=None)

Calculates the both sides (matrix-matrix or matrix-vector) product  $y.H @ A @ x$ , where  $A$  is the composed matrix represented by this decomposition.

**Parameters**

- x (numpy.ndarray or scipy.sparse.spmatrix) – Vector or matrix in the product  $y.H @ A @ x$ . It must hold  $self.n == x.shape[0]$ .
- y (numpy.ndarray or scipy.sparse.spmatrix) – Vector or matrix in the product  $y.H @ A @ x$ . It must hold  $self.n == y.shape[0]$ . optional, default: If y is not passed, x is used as y.
- dtype (numpy.dtype) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $y.H @ A @ x$ .

**Return type** numpy.ndarray or scipy.sparse.spmatrix

**matrix\_right\_side\_multiplication**(x, dtype=None)

Calculates the right side (matrix-matrix or matrix-vector) product  $A @ x$ , where  $A$  is the composed matrix represented by this decomposition.

**Parameters**

- x (numpy.ndarray or scipy.sparse.spmatrix) – Vector or matrix in the product in the matrix-matrix or matrix-vector  $A @ x$ . It must hold  $self.n == x.shape[0]$ .
- dtype (numpy.dtype) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $A @ x$ .

**Return type** numpy.ndarray or scipy.sparse.spmatrix

**n**

The dimension of the squared decomposed matrix.

**Type** int

**p**

The permutation vector.  $A[p[:, np.newaxis], p[np.newaxis, :]]$  is the matrix  $A$  permuted by the permutation of the decomposition

**Type** numpy.ndarray

**p\_inverse**

The permutation vector that undoes the permutation.

**Type** `numpy.ndarray`

**permute\_matrix(A)**  
Permutes a matrix by the permutation of the decomposition.

**Parameters** `A` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be permuted.

**Returns** The matrix `A` permuted by the permutation of the decomposition.

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**save(filename)**  
Saves this decomposition.

**Parameters** `filename` (`str`) – Where this decomposition should be saved.

**solve(b, dtype=None)**  
Solves the equation  $A x = b$  regarding  $x$ , where  $A$  is the composed matrix represented by this decomposition.

**Parameters**

- `b` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Right-hand side vector or matrix in equation  $A x = b$ . It must hold `self.n == b.shape[0]`.
- `dtype` (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** An  $x$  so that  $A x = b$ . The shape of  $x$  matches the shape of  $b$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**type\_str = 'LL'**  
The type of this decomposition represented as string.

**Type** `str`

**unpermute\_matrix(A)**  
Unpermutes a matrix permuted by the permutation of the decomposition.

**Parameters** `A` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be unpermuted.

**Returns** The matrix `A` unpermuted by the permutation of the decomposition.

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

## 4.2.2 LDL decomposition

**class** `matrix.decompositions.LDL_Decomposition(L=None, d=None, p=None)`

Bases: `matrix.decompositions.DecompositionBase`

A matrix decomposition where  $LDL^H$  is the decomposed (permuted) matrix.

$L$  is a lower triangle matrix with ones on the diagonal.  $D$  is a diagonal matrix. Only the diagonal values of  $D$  are stored.

**Parameters**

- `L` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix  $L$  of the decomposition. optional, If it is not set yet, it must be set later.

- **d** (`numpy.ndarray`) – The vector of the diagonal components of  $D$  of the decompositon. optional, If it is not set yet, it must be set later.
- **p** (`numpy.ndarray`) – The permutation vector used for the decomposition. This decomposition is of  $A[p[:, np.newaxis], p[np.newaxis, :]]$  where  $A$  is a matrix. optional, default: no permutation

**D**

The permutation matrix.

**Type** `scipy.sparse.dia_matrix`

**L**

The matrix  $L$  of the decomposition.

**Type** `numpy.matrix` or `scipy.sparse.spmatrix`

**LD**

A matrix whose diagonal values are the diagonal values of  $D$  and whose off-diagonal values are those of  $L$ .

**Type** `numpy.matrix` or `scipy.sparse.spmatrix`

**P**

The permutation matrix.  $P @ A @ P.T$  is the matrix  $A$  permuted by the permutation of the decomposition

**Type** `scipy.sparse.dok_matrix`

**append\_block\_decomposition(dec)**

Makes a new decomposition where this decompisition and the passed one are appended.

**Parameters** `dec` (`DecompositionBase`) – The decomposition that should be appended to this decomposition.

**Returns** `dec` – A new decomposition where this decompisition and the passed one are appended. This desomposition represents the first block and the passed one the second block.

**Return type** `DecompositionBase`

**as\_LDL\_DecompositionCompressed()****as\_LL\_Decomposition()****as\_any\_type(\*type\_strs, copy=False)**

Converts this decomposition to any of the passed types.

**Parameters**

- **\*type\_strs** (`str`) – The decomposition types to any of them this this decomposition is converted.
- **copy** (`bool`) – Whether the data of this decomposition should always be copied or only if needed.

**Returns** If the type of this decomposition is not in `type_strs`, a decomposition of type `type_str[0]` is returned which represents the same decomposed matrix as this decomposition. Otherwise this decomposition or a copy of it is returned, depending on `copy`.

**Return type** `matrix.decompositions.DecompositionBase`

**as\_same\_type(dec, copy=False)**

Converts the passed decompositions to the same type as this decomposition.

**Parameters**

- **dec** (`DecompositionBase`) – The decomposition that should be converted.

- **copy** (`bool`) – Whether the data of the decomposition that could be converted should always be copied or only if needed.

**Returns** If the type of the passed decomposition is not same type as this decomposition, a decomposition of this type is returned which represents the same decomposed matrix as the passed decomposition. Otherwise this decomposition or a copy of it is returned, depending on `copy`.

**Return type** `matrix.decompositions.DecompositionBase`

**as\_type** (`type_str, copy=False`)

Converts this decomposition to passed type.

#### Parameters

- **type\_str** (`str`) – The decomposition type to which this decomposition is converted.
- **copy** (`bool`) – Whether the data of this decomposition should always be copied or only if needed.

**Returns** If the type of this decomposition is not `type_str`, a decomposition of type `type_str` is returned which represents the same decomposed matrix as this decomposition. Otherwise this decomposition or a copy of it is returned, depending on `copy`.

**Return type** `matrix.decompositions.DecompositionBase`

**check\_finite** (`check_finite=True`)

Check if this is a decomposition representing a finite matrix.

**Parameters** `check_finite` (`bool`) – Whether to perform this check. default: True

**Raises** `matrix.errors.DecompositionNotFiniteError` – If this is a decomposition representing a non-finite matrix.

**check\_invertible()**

Check if this is a decomposition representing an invertible matrix.

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**composed\_matrix**

The composed matrix represented by this decomposition.

**Type** `numpy.matrix` or `scipy.sparse.spmatrix`

**copy()**

Copies this decomposition.

**Returns** A copy of this decomposition.

**Return type** `matrix.decompositions.DecompositionBase`

**d**

The diagonal vector of the matrix  $D$  of the decomposition.

**Type** `numpy.ndarray`

**inverse\_matrix\_both\_sides\_multiplication** (`x, y=None, dtype=None`)

Calculates the both sides (matrix-matrix or matrix-vector) product  $y.H @ B @ x$ , where  $B$  is the matrix inverse of the composed matrix represented by this decomposition.

#### Parameters

- **x** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ B @ x$ . It must hold `self.n == x.shape[0]`.

- **y** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ B @ x$ . It must hold `self.n == y.shape[0]`. optional, default: If `y` is not passed, `x` is used as `y`.
- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $y.H @ A @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

#### `inverse_matrix_right_side_multiplication(x, dtype=None)`

Calculates the right side (matrix-matrix or matrix-vector) product  $B @ x$ , where  $B$  is the matrix inverse of the composed matrix represented by this decomposition.

**Parameters**

- **x** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product in the matrix-matrix or matrix-vector  $B @ x$ . It must hold `self.n == x.shape[0]`.
- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $B @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

#### `is_almost_equal(other, rtol=0.0001, atol=1e-06)`

Whether this decomposition is close to passed decomposition.

**Parameters**

- **other** (`str`) – The decomposition which to compare to this decomposition.
- **rtol** (`float`) – The relative tolerance parameter.
- **atol** (`float`) – The absolute tolerance parameter.

**Returns** Whether this decomposition is close to passed decomposition.

**Return type** `bool`

#### `is_equal(other)`

Whether this decomposition is equal to passed decomposition.

**Parameters** **other** (`str`) – The decomposition which to compare to this decomposition.

**Returns** Whether this decomposition is equal to passed decomposition.

**Return type** `bool`

#### `is_finite()`

Returns whether this is a decomposition representing a finite matrix.

**Returns** Whether this is a decomposition representing a finite matrix.

**Return type** `bool`

#### `is_invertible()`

Returns whether this is a decomposition representing an invertible matrix.

**Returns** Whether this is a decomposition representing an invertible matrix.

**Return type** bool

**is\_permuted**

Whether this is a decompositon with permutation.

**Type** bool

**is\_positive\_definite()**

Returns whether this is a decomposition of a positive definite matrix.

**Returns** Whether this is a decomposition of a positive definite matrix.

**Return type** bool

**is\_positive\_semidefinite()**

Returns whether this is a decomposition of a positive semi-definite matrix.

**Returns** Whether this is a decomposition of a positive semi-definite matrix.

**Return type** bool

**is\_singular()**

Returns whether this is a decomposition representing a singular matrix.

**Returns** Whether this is a decomposition representing a singular matrix.

**Return type** bool

**is\_sparse()**

Returns whether this is a decomposition of a sparse matrix.

**Returns** Whether this is a decomposition of a sparse matrix.

**Return type** bool

**is\_type**(*type\_str*)

Whether this decomposition is of the passed type.

**Parameters** `type_str`(*str*) – The decomposition type according to which is checked.

**Returns** Whether this is a decomposition of the passed type.

**Return type** bool

**load**(*filename*)

Loads a decomposition of this type.

**Parameters** `filename`(*str*) – Where the decomposition is saved.

**Raises**

- `FileNotFoundException` – If the files are not found in the passed directory.
- `DecompositionInvalidDecompositionTypeFile` – If the files contains another decomposition type.

**matrix\_both\_sides\_multiplication**(*x*, *y=None*, *dtype=None*)

Calculates the both sides (matrix-matrix or matrix-vector) product  $y.H @ A @ x$ , where  $A$  is the composed matrix represented by this decomposition.

**Parameters**

- `x` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ A @ x$ . It must hold `self.n == x.shape[0]`.

- **y** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ A @ x$ . It must hold `self.n == y.shape[0]`. optional, default: If y is not passed, x is used as y.
- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $y.H @ A @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

### `matrix_right_side_multiplication(x, dtype=None)`

Calculates the right side (matrix-matrix or matrix-vector) product  $A @ x$ , where  $A$  is the composed matrix represented by this decomposition.

#### Parameters

- **x** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product in the matrix-matrix or matrix-vector  $A @ x$ . It must hold `self.n == x.shape[0]`.
- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $A @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**n**

The dimension of the squared decomposed matrix.

**Type** `int`

**p**

The permutation vector.  $A[p[:, np.newaxis], p[np.newaxis, :]]$  is the matrix  $A$  permuted by the permutation of the decomposition

**Type** `numpy.ndarray`

**p\_inverse**

The permutation vector that undoes the permutation.

**Type** `numpy.ndarray`

**permute\_matrix(A)**

Permutes a matrix by the permutation of the decomposition.

**Parameters** **A** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be permuted.

**Returns** The matrix  $A$  permuted by the permutation of the decomposition.

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**save(filename)**

Saves this decomposition.

**Parameters** **filename** (`str`) – Where this decomposition should be saved.

**solve(b, dtype=None)**

Solves the equation  $A x = b$  regarding  $x$ , where  $A$  is the composed matrix represented by this decomposition.

#### Parameters

- **b** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Right-hand side vector or matrix in equation  $A x = b$ . It must hold `self.n == b.shape[0]`.

- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** An  $x$  so that  $A x = b$ . The shape of  $x$  matches the shape of  $b$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**type\_str** = 'LDL'

The type of this decomposition represented as string.

**Type** `str`

**unpermute\_matrix** ( $A$ )

Unpermutes a matrix permuted by the permutation of the decomposition.

**Parameters**  $A$  (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be unpermuted.

**Returns** The matrix  $A$  unpermuted by the permutation of the decomposition.

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

### 4.2.3 LDL decomposition compressed

**class** `matrix.decompositions.LDL_DecompositionCompressed` ( $LD=None$ ,  $p=None$ )

Bases: `matrix.decompositions.DecompositionBase`

A matrix decomposition where  $LDL^H$  is the decomposed (permuted) matrix.

$L$  is a lower triangle matrix with ones on the diagonal.  $D$  is a diagonal matrix.  $L$  and  $D$  are stored in one matrix whose diagonal values are the diagonal values of  $D$  and whose off-diagonal values are those of  $L$ .

#### Parameters

- **LD** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – A matrix whose diagonal values are the diagonal values of  $D$  and whose off-diagonal values are those of  $L$ . optional, If it is not set yet, it must be set later.
- **p** (`numpy.ndarray`) – The permutation vector used for the decomposition. This decomposition is of  $A[p[:, np.newaxis], p[np.newaxis, :]]$  where  $A$  is a matrix. optional, default: no permutation

**D**

The permutation matrix.

**Type** `scipy.sparse.dia_matrix`

**L**

The matrix  $L$  of the decomposition.

**Type** `numpy.matrix` or `scipy.sparse.spmatrix`

**LD**

A matrix whose diagonal values are the diagonal values of  $D$  and whose off-diagonal values are those of  $L$ .

**Type** `numpy.matrix` or `scipy.sparse.spmatrix`

**P**

The permutation matrix.  $P @ A @ P.T$  is the matrix  $A$  permuted by the permutation of the decomposition

**Type** `scipy.sparse.dok_matrix`

**append\_block\_decomposition**(*dec*)

Makes a new decomposition where this decompositon and the passed one are appended.

**Parameters** `dec` (`DecompositionBase`) – The decomposition that should be appended to this decomposition.

**Returns** `dec` – A new decomposition where this decompositon and the passed one are appended. This desomposition represents the first block and the passed one the second block.

**Return type** `DecompositionBase`

**as\_LDL\_Decomposition**()

**as\_any\_type**(\**type\_strs*, *copy=False*)

Converts this decomposition to any of the passed types.

**Parameters**

- `*type_strs` (`str`) – The decomposition types to any of them this this decomposition is converted.
- `copy` (`bool`) – Whether the data of this decomposition should always be copied or only if needed.

**Returns** If the type of this decomposition is not in *type\_strs*, a decomposition of type *type\_str[0]* is returned which represents the same decomposed matrix as this decomposition. Otherwise this decomposition or a copy of it is returned, depending on *copy*.

**Return type** `matrix.decompositions.DecompositionBase`

**as\_same\_type**(*dec*, *copy=False*)

Converts the passed decompositions to the same type as this decomposition.

**Parameters**

- `doc` (`DecompositionBase`) – The decomposition that should be converted.
- `copy` (`bool`) – Whether the data of the decomposition that could be converted should always be copied or only if needed.

**Returns** If the type of the passed decomposition is not same type as this decomposition, a decomposition of this type is returned which represents the same decomposed matrix as the passed decomposition. Otherwise this decomposition or a copy of it is returned, depending on *copy*.

**Return type** `matrix.decompositions.DecompositionBase`

**as\_type**(*type\_str*, *copy=False*)

Converts this decomposition to passed type.

**Parameters**

- `type_str` (`str`) – The decomposition type to which this decomposition is converted.
- `copy` (`bool`) – Whether the data of this decomposition should always be copied or only if needed.

**Returns** If the type of this decomposition is not *type\_str*, a decomposition of type *type\_str* is returned which represents the same decomposed matrix as this decomposition. Otherwise this decomposition or a copy of it is returned, depending on *copy*.

**Return type** `matrix.decompositions.DecompositionBase`

**check\_finite**(*check\_finite=True*)

Check if this is a decomposition representing a finite matrix.

**Parameters** `check_finite` (`bool`) – Whether to perform this check. default: True

**Raises** `matrix.errors.DecompositionNotFiniteError` – If this is a decomposition representing a non-finite matrix.

**check\_invertible()**

Check if this is a decomposition representing an invertible matrix.

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**composed\_matrix**

The composed matrix represented by this decomposition.

**Type** `numpy.matrix` or `scipy.sparse.spmatrix`

**copy()**

Copies this decomposition.

**Returns** A copy of this decomposition.

**Return type** `matrix.decompositions.DecompositionBase`

**d**

The diagonal vector of the matrix  $D$  of the decomposition.

**Type** `numpy.ndarray`

**inverse\_matrix\_both\_sides\_multiplication**(*x, y=None, dtype=None*)

Calculates the both sides (matrix-matrix or matrix-vector) product  $y.H @ B @ x$ , where  $B$  is the matrix inverse of the composed matrix represented by this decomposition.

**Parameters**

- **x** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ B @ x$ . It must hold `self.n == x.shape[0]`.
- **y** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ B @ x$ . It must hold `self.n == y.shape[0]`. optional, default: If y is not passed, x is used as y.
- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $y.H @ A @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**inverse\_matrix\_right\_side\_multiplication**(*x, dtype=None*)

Calculates the right side (matrix-matrix or matrix-vector) product  $B @ x$ , where  $B$  is the matrix inverse of the composed matrix represented by this decomposition.

**Parameters**

- **x** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product in the matrix-matrix or matrix-vector  $B @ x$ . It must hold `self.n == x.shape[0]`.
- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $B @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**is\_almost\_equal** (`other, rtol=0.0001, atol=1e-06`)

Whether this decomposition is close to passed decomposition.

**Parameters**

- **other** (`str`) – The decomposition which to compare to this decomposition.
- **rtol** (`float`) – The relative tolerance parameter.
- **atol** (`float`) – The absolute tolerance parameter.

**Returns** Whether this decomposition is close to passed decomposition.

**Return type** `bool`

**is\_equal** (`other`)

Whether this decomposition is equal to passed decomposition.

**Parameters** **other** (`str`) – The decomposition which to compare to this decomposition.

**Returns** Whether this decomposition is equal to passed decomposition.

**Return type** `bool`

**is\_finite** ()

Returns whether this is a decomposition representing a finite matrix.

**Returns** Whether this is a decomposition representing a finite matrix.

**Return type** `bool`

**is\_invertible** ()

Returns whether this is a decomposition representing an invertible matrix.

**Returns** Whether this is a decomposition representing an invertible matrix.

**Return type** `bool`

**is\_permuted**

Whether this is a decompositon with permutation.

**Type** `bool`

**is\_positive\_definite** ()

Returns whether this is a decomposition of a positive definite matrix.

**Returns** Whether this is a decomposition of a positive definite matrix.

**Return type** `bool`

**is\_positive\_semidefinite** ()

Returns whether this is a decomposition of a positive semi-definite matrix.

**Returns** Whether this is a decomposition of a positive semi-definite matrix.

**Return type** `bool`

**is\_singular** ()

Returns whether this is a decomposition representing a singular matrix.

**Returns** Whether this is a decomposition representing a singular matrix.

**Return type** `bool`

**is\_sparse()**

Returns whether this is a decomposition of a sparse matrix.

**Returns** Whether this is a decomposition of a sparse matrix.

**Return type** `bool`

**is\_type(type\_str)**

Whether this decomposition is of the passed type.

**Parameters** `type_str` – The decomposition type according to which is checked.

**Returns** Whether this is a decomposition of the passed type.

**Return type** `bool`

**load(filename)**

Loads a decomposition of this type.

**Parameters** `filename` – Where the decomposition is saved.

**Raises**

- `FileNotFoundException` – If the files are not found in the passed directory.
- `DecompositionInvalidDecompositionTypeFile` – If the files contains another decomposition type.

**matrix\_both\_sides\_multiplication(x, y=None, dtype=None)**

Calculates the both sides (matrix-matrix or matrix-vector) product  $y.H @ A @ x$ , where  $A$  is the composed matrix represented by this decomposition.

**Parameters**

- `x` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ A @ x$ . It must hold `self.n == x.shape[0]`.
- `y` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ A @ x$ . It must hold `self.n == y.shape[0]`. optional, default: If `y` is not passed, `x` is used as `y`.
- `dtype` (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $y.H @ A @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**matrix\_right\_side\_multiplication(x, dtype=None)**

Calculates the right side (matrix-matrix or matrix-vector) product  $A @ x$ , where  $A$  is the composed matrix represented by this decomposition.

**Parameters**

- `x` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product in the matrix-matrix or matrix-vector  $A @ x$ . It must hold `self.n == x.shape[0]`.
- `dtype` (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $A @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**n**

The dimension of the squared decomposed matrix.

**Type** `int`

**p**

The permutation vector.  $A[p[:, np.newaxis], p[np.newaxis, :]]$  is the matrix  $A$  permuted by the permutation of the decomposition

**Type** `numpy.ndarray`

**p\_inverse**

The permutation vector that undoes the permutation.

**Type** `numpy.ndarray`

**permute\_matrix(A)**

Permutes a matrix by the permutation of the decomposition.

**Parameters** `A` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be permuted.

**Returns** The matrix  $A$  permuted by the permutation of the decomposition.

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**save(filename)**

Saves this decomposition.

**Parameters** `filename` (`str`) – Where this decomposition should be saved.

**solve(b, dtype=None)**

Solves the equation  $A x = b$  regarding  $x$ , where  $A$  is the composed matrix represented by this decomposition.

**Parameters**

- `b` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Right-hand side vector or matrix in equation  $A x = b$ . It must hold `self.n == b.shape[0]`.
- `dtype` (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** An  $x$  so that  $A x = b$ . The shape of  $x$  matches the shape of  $b$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**type\_str = 'LDL\_compressed'**

The type of this decomposition represented as string.

**Type** `str`

**unpermute\_matrix(A)**

Unpermutes a matrix permuted by the permutation of the decomposition.

**Parameters** `A` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be unpermuted.

**Returns** The matrix  $A$  unpermuted by the permutation of the decomposition.

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

## 4.2.4 Base decomposition

**class** `matrix.decompositions.DecompositionBase` (`p=None`)  
Bases: `object`

A matrix decomposition.

This class is a base class for all other matrix decompositions.

**Parameters** `p` (`numpy.ndarray`) – The permutation vector used for the decomposition. This decomposition is of  $A[p[:, np.newaxis], p[np.newaxis, :]]$  where  $A$  is a matrix. optional, default: no permutation

**P**

The permutation matrix.  $P @ A @ P.T$  is the matrix  $A$  permuted by the permutation of the decomposition

**Type** `scipy.sparse.dok_matrix`

**append\_block\_decomposition** (`dec`)

Makes a new decomposition where this decomposition and the passed one are appended.

**Parameters** `dec` (`DecompositionBase`) – The decomposition that should be appended to this decomposition.

**Returns** `dec` – A new decomposition where this decomposition and the passed one are appended.  
This decomposition represents the first block and the passed one the second block.

**Return type** `DecompositionBase`

**as\_any\_type** (\*`type_strs`, `copy=False`)

Converts this decomposition to any of the passed types.

**Parameters**

- `*type_strs` (`str`) – The decomposition types to any of them this decomposition is converted.
- `copy` (`bool`) – Whether the data of this decomposition should always be copied or only if needed.

**Returns** If the type of this decomposition is not in `type_strs`, a decomposition of type `type_str[0]` is returned which represents the same decomposed matrix as this decomposition. Otherwise this decomposition or a copy of it is returned, depending on `copy`.

**Return type** `matrix.decompositions.DecompositionBase`

**as\_same\_type** (`dec`, `copy=False`)

Converts the passed decompositions to the same type as this decomposition.

**Parameters**

- `dec` (`DecompositionBase`) – The decomposition that should be converted.
- `copy` (`bool`) – Whether the data of the decomposition that could be converted should always be copied or only if needed.

**Returns** If the type of the passed decomposition is not same type as this decomposition, a decomposition of this type is returned which represents the same decomposed matrix as the passed decomposition. Otherwise this decomposition or a copy of it is returned, depending on `copy`.

**Return type** `matrix.decompositions.DecompositionBase`

**as\_type** (`type_str`, `copy=False`)

Converts this decomposition to passed type.

## Parameters

- **type\_str** (`str`) – The decomposition type to which this decomposition is converted.
- **copy** (`bool`) – Whether the data of this decomposition should always be copied or only if needed.

**Returns** If the type of this decomposition is not `type_str`, a decomposition of type `type_str` is returned which represents the same decomposed matrix as this decomposition. Otherwise this decomposition or a copy of it is returned, depending on `copy`.

**Return type** `matrix.decompositions.DecompositionBase`

**check\_finite** (`check_finite=True`)

Check if this is a decomposition representing a finite matrix.

**Parameters** `check_finite` (`bool`) – Whether to perform this check. default: True

**Raises** `matrix.errors.DecompositionNotFiniteError` – If this is a decomposition representing a non-finite matrix.

**check\_invertible** ()

Check if this is a decomposition representing an invertible matrix.

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**composed\_matrix**

The composed matrix represented by this decomposition.

**Type** `numpy.matrix` or `scipy.sparse.spmatrix`

**copy** ()

Copies this decomposition.

**Returns** A copy of this decomposition.

**Return type** `matrix.decompositions.DecompositionBase`

**inverse\_matrix\_both\_sides\_multiplication** (`x, y=None, dtype=None`)

Calculates the both sides (matrix-matrix or matrix-vector) product  $y.H @ B @ x$ , where  $B$  is the matrix inverse of the composed matrix represented by this decomposition.

## Parameters

- **x** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ B @ x$ . It must hold `self.n == x.shape[0]`.
- **y** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ B @ x$ . It must hold `self.n == y.shape[0]`. optional, default: If y is not passed, x is used as y.
- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $y.H @ A @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**inverse\_matrix\_right\_side\_multiplication** (`x, dtype=None`)

Calculates the right side (matrix-matrix or matrix-vector) product  $B @ x$ , where  $B$  is the matrix inverse of the composed matrix represented by this decomposition.

## Parameters

- **x** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product in the matrix-matrix or matrix-vector  $B @ x$ . It must hold `self.n == x.shape[0]`.
- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $B @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**is\_almost\_equal** (`other, rtol=0.0001, atol=1e-06`)

Whether this decomposition is close to passed decomposition.

## Parameters

- **other** (`str`) – The decomposition which to compare to this decomposition.
- **rtol** (`float`) – The relative tolerance parameter.
- **atol** (`float`) – The absolute tolerance parameter.

**Returns** Whether this decomposition is close to passed decomposition.

**Return type** `bool`

**is\_equal** (`other`)

Whether this decomposition is equal to passed decomposition.

**Parameters** **other** (`str`) – The decomposition which to compare to this decomposition.

**Returns** Whether this decomposition is equal to passed decomposition.

**Return type** `bool`

**is\_finite** ()

Returns whether this is a decomposition representing a finite matrix.

**Returns** Whether this is a decomposition representing a finite matrix.

**Return type** `bool`

**is\_invertible** ()

Returns whether this is a decomposition representing an invertible matrix.

**Returns** Whether this is a decomposition representing an invertible matrix.

**Return type** `bool`

**is\_permuted**

Whether this is a decompositon with permutation.

**Type** `bool`

**is\_positive\_definite** ()

Returns whether this is a decomposition of a positive definite matrix.

**Returns** Whether this is a decomposition of a positive definite matrix.

**Return type** `bool`

**is\_positive\_semidefinite** ()

Returns whether this is a decomposition of a positive semi-definite matrix.

**Returns** Whether this is a decomposition of a positive semi-definite matrix.

**Return type** bool

**is\_singular()**

Returns whether this is a decomposition representing a singular matrix.

**Returns** Whether this is a decomposition representing a singular matrix.

**Return type** bool

**is\_sparse()**

Returns whether this is a decomposition of a sparse matrix.

**Returns** Whether this is a decomposition of a sparse matrix.

**Return type** bool

**is\_type(type\_str)**

Whether this decomposition is of the passed type.

**Parameters** `type_str` – The decomposition type according to which is checked.

**Returns** Whether this is a decomposition of the passed type.

**Return type** bool

**load(filename)**

Loads a decomposition of this type.

**Parameters** `filename` – Where the decomposition is saved.

**Raises**

- `FileNotFoundException` – If the files are not found in the passed directory.
- `DecompositionInvalidDecompositionTypeFile` – If the files contains another decomposition type.

**matrix\_both\_sides\_multiplication(x, y=None, dtype=None)**

Calculates the both sides (matrix-matrix or matrix-vector) product  $y.H @ A @ x$ , where  $A$  is the composed matrix represented by this decomposition.

**Parameters**

- `x` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ A @ x$ . It must hold `self.n == x.shape[0]`.
- `y` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product  $y.H @ A @ x$ . It must hold `self.n == y.shape[0]`. optional, default: If `y` is not passed, `x` is used as `y`.
- `dtype` (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $y.H @ A @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**matrix\_right\_side\_multiplication(x, dtype=None)**

Calculates the right side (matrix-matrix or matrix-vector) product  $A @ x$ , where  $A$  is the composed matrix represented by this decomposition.

**Parameters**

- `x` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Vector or matrix in the product in the matrix-matrix or matrix-vector  $A @ x$ . It must hold `self.n == x.shape[0]`.

- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** The result of  $A @ x$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**n**

The dimension of the squared decomposed matrix.

**Type** `int`

**p**

The permutation vector.  $A[p[:, np.newaxis], p[np.newaxis, :]]$  is the matrix  $A$  permuted by the permutation of the decomposition

**Type** `numpy.ndarray`

**p\_inverse**

The permutation vector that undoes the permutation.

**Type** `numpy.ndarray`

**permute\_matrix(A)**

Permutes a matrix by the permutation of the decomposition.

**Parameters** **A** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be permuted.

**Returns** The matrix  $A$  permuted by the permutation of the decomposition.

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**save(filename)**

Saves this decomposition.

**Parameters** **filename** (`str`) – Where this decomposition should be saved.

**solve(b, dtype=None)**

Solves the equation  $A x = b$  regarding  $x$ , where  $A$  is the composed matrix represented by this decomposition.

**Parameters**

- **b** (`numpy.ndarray` or `scipy.sparse.spmatrix`) – Right-hand side vector or matrix in equation  $A x = b$ . It must hold `self.n == b.shape[0]`.
- **dtype** (`numpy.dtype`) – Type to use in computation. optional, default: Determined by the method.

**Returns** An  $x$  so that  $A x = b$ . The shape of  $x$  matches the shape of  $b$ .

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

**Raises** `matrix.errors.DecompositionSingularError` – If this is a decomposition representing a singular matrix.

**type\_str = 'base'**

The type of this decomposition represented as string.

**Type** `str`

**unpermute\_matrix(A)**

Unpermutes a matrix permuted by the permutation of the decomposition.

**Parameters** `A` (`numpy.ndarray` or `scipy.sparse.spmatrix`) – The matrix that should be unpermuted.

**Returns** The matrix A unpermuted by the permutation of the decomposition.

**Return type** `numpy.ndarray` or `scipy.sparse.spmatrix`

## 4.3 Errors

This is an overview about the exceptions that could arise in this library. They are available in `matrix.errors`:

The following exception is the base exception from which all other exceptions in this package are derived:

### 4.3.1 BaseError

```
exception matrix.errors.BaseError(message)
Bases: Exception
```

This is the base exception for all exceptions in this package.

If a matrix has an invalid properties, the following exceptions can occur:

### 4.3.2 MatrixError

```
exception matrix.errors.MatrixError(matrix, message=None)
Bases: matrix.errors.BaseError

An exception related to a matrix.
```

### 4.3.3 MatrixNotSquareError

```
exception matrix.errors.MatrixNotSquareError(matrix)
Bases: matrix.errors.MatrixError

A matrix is not a square matrix although a square matrix is required.
```

### 4.3.4 MatrixNotFiniteError

```
exception matrix.errors.MatrixNotFiniteError(matrix)
Bases: matrix.errors.MatrixError

A matrix has non-finite entries although a finite matrix is required.
```

### 4.3.5 MatrixSingularError

```
exception matrix.errors.MatrixSingularError(matrix)
Bases: matrix.errors.MatrixError

A matrix is singular although an invertible matrix is required.
```

### 4.3.6 MatrixNotHermitianError

```
exception matrix.errors.MatrixNotHermitianError(matrix, i=None, j=None)
Bases: matrix.errors.MatrixError
```

A matrix is not Hermitian although a Hermitian matrix is required.

### 4.3.7 MatrixComplexDiagonalValueError

```
exception matrix.errors.MatrixComplexDiagonalValueError(matrix, i=None)
Bases: matrix.errors.MatrixNotHermitianError
```

A matrix has complex diagonal values although real diagonal values are required.

If a desired decomposition is not computable, the following exceptions can be raised:

### 4.3.8 NoDecompositionPossibleError

```
exception matrix.errors.NoDecompositionPossibleError(base, desired_type)
Bases: matrix.errors.BaseError
```

It is not possible to calculate a desired matrix decomposition.

### 4.3.9 NoDecompositionPossibleWithProblematicSubdecompositionError

```
exception matrix.errors.NoDecompositionPossibleWithProblematicSubdecompositionError(base,
desired_type,
problematic_lead,
subdecomposition,
po-
si-
tion=None)
Bases: matrix.errors.NoDecompositionPossibleError
```

It is not possible to calculate a desired matrix decomposition. Only a subdecomposition could be calculated

### 4.3.10 NoDecompositionPossibleTooManyEntriesError

```
exception matrix.errors.NoDecompositionPossibleTooManyEntriesError(matrix,
desired_type)
Bases: matrix.errors.NoDecompositionPossibleError
```

The decomposition is not possible for this matrix because it would have too many entries.

### 4.3.11 NoDecompositionConversionImplementedError

```
exception matrix.errors.NoDecompositionConversionImplementedError(decomposition,
de-
sired_type)
```

Bases: *matrix.errors.NoDecompositionPossibleError*

A decomposition conversion is not implemented for this type.

If the matrix, represented by a decomposition, has an invalid characteristic, the following exceptions can occur:

### 4.3.12 DecompositionError

```
exception matrix.errors.DecompositionError(decomposition, message=None)
```

Bases: *matrix.errors.BaseError*

An exception related to a decomposition.

### 4.3.13 DecompositionNotFiniteError

```
exception matrix.errors.DecompositionNotFiniteError(decomposition)
```

Bases: *matrix.errors.DecompositionError*

A decomposition of a matrix has non-finite entries although a finite matrix is required.

### 4.3.14 DecompositionSingularError

```
exception matrix.errors.DecompositionSingularError(decomposition)
```

Bases: *matrix.errors.DecompositionError*

A decomposition represents a singular matrix although a non-singular matrix is required.

If a decomposition could not be loaded from a file, the following exceptions can be raised:

### 4.3.15 DecompositionInvalidFile

```
exception matrix.errors.DecompositionInvalidFile(filename)
```

Bases: *matrix.errors.DecompositionError, OSError*

An attempt was made to load a decomposition from an invalid file.

### 4.3.16 DecompositionInvalidDecompositionTypeFile

```
exception matrix.errors.DecompositionInvalidDecompositionTypeFile(filename,
type_file,
type_needed)
```

Bases: *matrix.errors.DecompositionInvalidFile*

An attempt was made to load a decomposition from a file in which another decomposition type is stored.

If the computation needs more iterations than the maximal number of iterations, the following exception occurs:

### 4.3.17 TooManyIterationsError

```
exception matrix.errors.TooManyIterationsError(message=None, iteration=None, result=None)
```

Bases: *matrix.errors.BaseError*

Too many iterations are needed for the calculation.

## 4.4 Changelog

### 4.4.1 1.2

- Several functions to solve nearness problems (nearest symmetric matrix, nearest skew symmetric matrix, nearest positive semidefinite matrix, nearest correlation matrix) added.
- Significant speedup of approximation algorithms in *matrix.approximation.positive\_semidefinite.Reimer*.
- Overflow handling improved in several functions.
- Decompositions in *matrix.decompositions* now have an *append\_block\_decomposition* and *as\_same\_type* method and their multiplication methods have now a *dtype* argument.
- The approximation algorithms in *matrix.approximation.positive\_semidefinite.Reimer* now have a *min\_abs\_value\_L* argument.

### 4.4.2 1.1

- Positive semidefinite approximation algorithms of GMW and SE type have been added.
- Permutation method, with numerical stability as main focus, has been added to positive semidefinite approximation algorithm.
- Positive semidefinite approximation algorithm are moved into separate package. (*matrix.approximate* to *matrix.approximation.positive\_semidefinite*)

### 4.4.3 1.0.1

- Approximation functions now also work if an overflows occurs.
- NumPys matrix is avoided because it is deprecated now.

### 4.4.4 1.0

- Approximation functions are slightly faster now.
- Better overflow handling is now used in approximation functions.
- Prebuild html documentation are now included.
- Function for approximating a matrix by a positive semidefinite matrix (*matrix.approximate.positive\_semidefinite\_matrix*) has been removed.

## **4.4.5 0.8**

- Approximation functions have been replaced by more sophisticated approximation functions.
- Explicit function for approximating a matrix by a positive (semi)definite matrix has been added.
- Universal save and load functions have been added.
- Decompositions have obtained `is_equal` and `is_almost_equal` methods.
- Functions to multiply the matrix, represented by a decomposition, or its inverse with a matrix or a vector have been added.
- It is now possible to pass permutation vectors to approximate and decompose methods.

## **4.4.6 0.7**

- Lineare systems associated to matrices or decompositions can now be solved.
- Invertibility of matrices and decompositions can now be examined.
- Decompositions can now be examined to see if they contain only finite values.

## **4.4.7 0.6**

- Decompositions are now saveable and loadable.

## **4.4.8 0.5**

- Matrices can now be approximated by decompositions.

## **4.4.9 0.4**

- Positive definiteness and positive semi-definiteness of matrices and decompositions can now be examined.

## **4.4.10 0.3**

- Dense and sparse matrices are now decomposable into several types (LL, LDL, LDL compressed).

## **4.4.11 0.2**

- Decompositons are now convertable to other decompositon types.
- Decompositions are now comparable.

## **4.4.12 0.1**

- Several decompositions types (LL, LDL, LDL compressed) have been added.
- Several permutation capabilities have been added.

## 4.5 Requirements

- Python >= 3.7
- NumPy >= 1.15
- SciPy >= 0.19
- scikit-sparse >= 0.4.2 (only for sparse matrix support)



# CHAPTER 5

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## Indices and tables

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# CHAPTER 6

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## Copyright

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